NONSEPARATED FILTRATION OF A SUSPENSION OF VARIABLE CONCENTRATION

V. Z. Kochmarskii

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This article examines effects which occur due the time-varying inlet concentration of impurities in a suspension. It is found that the time during which charging of a filter exerts a protective effect depends on the method by which this time is determined. The dependence of filtration characteristics on the quality of regeneration of the filter medium is established.

Mathematical modeling of the bulk filtration of suspensions on porous beds is an integral part of constructing a model of any water purification system or commercial system for separating multiphase media. As a rule, the composition of the suspension varies over time during the operation of such systems. Nevertheless, it has been assumed in well-known studies [1-4] that the inlet concentration of impurities in filtration is constant. The main filtration characteristics used in optimization calculations are calculated with this assumption. At the same time, quantities such as filtration efficiency, filtering time, the working capacity of the filter, and other characteristics determined for a constant inlet concentration of impurities need to be refined in the case of a variable concentration. Mathematically, this means that new degrees of freedom characterizing the filtration process must be considered — the frequency and amplitude of changes in impurity concentration and the relationship between the fluctuating and constant components of this concentration.

In fact, in the case of an arbitrary relation for inlet concentration C(t), it can be represented in the form

$$C(t) = \langle c \rangle_t + c(t) \equiv \langle c \rangle_t [1 + \beta f(t)], \tag{1}$$

$$\langle c \rangle_t = \frac{1}{t}\int_0^t C(t)\,dt;\ c(t) = c_{osc} f(t);\ \beta = \frac{c_{osc}}{\langle c \rangle_t}. \tag{2}$$

Averaging in (2) is performed over a time interval t which is comparable to the effective period of operation of the filter.

To determine the main effects of a time-varying inlet concentration, we will examine a filtration model of the type developed by Yu. M. Shekhtman [1]. This model is convenient because its solutions can be obtained analytically in closed form. Within the framework of the Shekhtman model, filtration is described by the equations

$$\begin{gathered}(\varepsilon C + \rho)_t + vC_x = 0;\quad \rho_t = bvC(1 - \rho/\rho_{cr});\\ C(x,\ t)|_{x=0} = C_0(t);\quad \rho(x,\ t)|_{t=0} = \rho_i(x).\end{gathered} \tag{3}$$

It is assumed that the filter is axially symmetric relative to OX, while the inlet section of the filter element (bed) coincides with the coordinate origin.

Model equations (3) differ from those used in [1] in that the term $(\varepsilon C(x,\ t))_t$, describing lag effects important in the case of variable concentration $C_0(t)$, is kept in the first equation. It should be noted that particle separation processes are not considered in (3). In fact, if C(x, t) = 0, then in accordance with (3) $\rho_t = 0$. Thus, no residue is washed out. As a result, (3) adequately describes only those filtration regimes in which deposition is dominant.

Imposing the condition that the porosity of the bed change little over time ($\varepsilon(x,\ t) \simeq \varepsilon(x)$), we obtain solutions to system (3) in the following manner.

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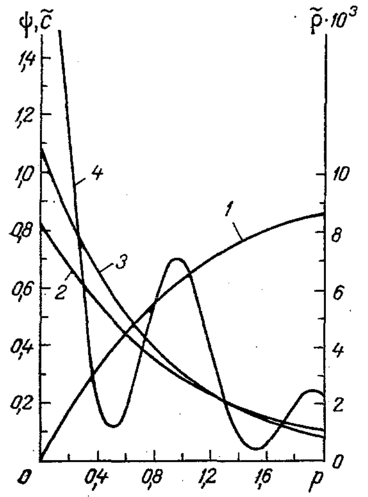

Fig. 1

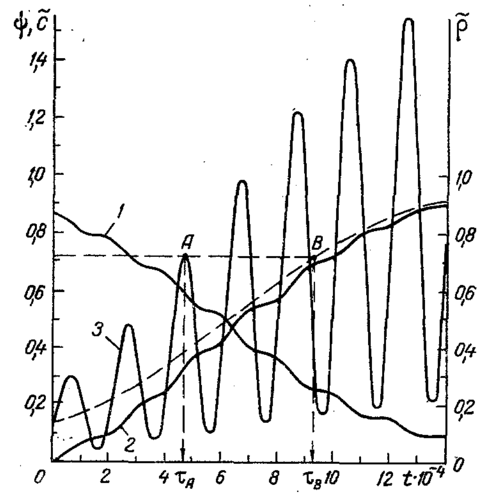

Fig. 2

Fig. 1. The relations $\psi(p)$, $\tilde{\rho}(p)$ and $\tilde{c}(p)$ (curves 1, 2 and 3, 4) calculated from Eqs. (8a) and (8b) for $\alpha = 10^{-3}$; $\beta = 0.8$; $\gamma = 8\cdot10^{-4}$; $t = 270$ sec; $b = 3.0\ \text{sec}^{-1}$; $v = 10^{-2}$ m/sec; $\varepsilon = 0.4$; for curve 3, $Q = 8.0$; for 4, $Q = 0.4$.

Fig. 2. The relations $\psi(q)$, $\tilde{\rho}(q)$ and $\tilde{c}(q)$ for $p = 2.0$; $Q = 6\cdot10^2$ (curves 1, 2, and 3); the values of the remaining quantities are the same as in Fig. 1. The quantities τ_A and τ_B are the periods of effective filtration determined from the amplitude and mean value of $\tilde{c}(p, q)$; the dashed line corresponds to $\tilde{c}(q)$ for the averaged inlet concentration.

By making the substitutions

$$t = \frac{1}{bv}\left[q + \int_0^{bx} \varepsilon(y)\,dy\right];\ x = p/b;$$
$$\tilde{c} = \frac{C}{\langle c\rangle_t};\ \tilde{\rho} = \frac{\rho}{\rho_{cr}};\ \alpha = \frac{\langle c\rangle_t}{\rho_{cr}} \tag{4}$$

we simplify system (3) to the form

$$(\ln\tilde{c})_{pq} - \tilde{\rho}_q = 0;\ \tilde{\rho}_q + \tilde{c}_p = 0;\ \tilde{c}|_{p=0} = 1 + \beta f(q);\ \tilde{\rho}|_{q\in L} = \tilde{\rho}_i(p/b). \tag{5}$$

In Eqs. (5), the line L is determined by the relation

$$q + \int_0^p \varepsilon(x/b)\,dx = 0. \tag{6}$$

On the basis of (5), we find for $(p, q)\in L$

$$\tilde{c}_L(p, q) = [1 + \beta f(q)]\exp\left[-p + \int_0^p \tilde{\rho}_i(x/b)\,dx\right]. \tag{7}$$

Equation (7) describes the law of change in impurity concentration with movement of the concentration front along the bed immediately after the filter begins operation.

Then using (5) and (6) and the boundary and initial condition $\tilde{c}|_{p=0}$ and $\tilde{\rho}|_{q\in L}$, we find exact solutions of (5):

$$\tilde{c}(p, q) = \frac{[1 + \beta f(q)]\exp[\alpha\varphi(p, q)]}{\exp\left[p - \int_0^p \tilde{\rho}_i(x/b)\,dx\right] + \exp[\alpha\varphi(p, q)] - 1}; \tag{8a}$$

$$\tilde{\rho}(p, q) = \tilde{\rho}_i(p) + \frac{[1 - \tilde{\rho}_i(p)] \exp[\alpha\varphi(p, q)]}{\exp\left[p - \int_0^p \tilde{\rho}_i(x/b)\, dx\right] + \exp[\alpha\varphi(p, q)] - 1}; \tag{8b}$$

$$\varphi(p, q) = q + \mu(p) + \beta \int_{\mu(p)}^{q} f(x)\, dx; \; \mu(p) = \int_0^p \varepsilon(x/b)\, dx. \tag{9}$$

The following conditions must be satisfied for effective beds

$$\alpha \ll 1; \; \gamma = \alpha\beta = \frac{c_{osc}}{\rho_{cr}} \ll 1. \tag{10}$$

Solution (8) differ from the well-known solutions in [1] in the presence of the terms

$$\int_0^p \varepsilon(x/b)\, dx; \; \beta \int_{\mu(p)}^{q} f(x)\, dx; \; \int_0^p \tilde{\rho}_i(x/b)\, dx,$$

which consider lag effects, oscillations, and the reduction in the filtering quality of the bed due to the initial distribution of the residue (incomplete regeneration of the bed).

We complete regeneration of the bed, $\rho_i(x) = 0$. This, the term $\int_0^p \tilde{\rho}_i(x/b)\, dx$ in (8) can be ignored if regeneration of the bed is incomplete. In this case, the term $\int_0^p \tilde{\rho}_i(x/b)\, dx$ is important for calculations of the filtering effect and determination of the length of the filtration cycle and filter efficiency.

Let us examine the effect of oscillations of inlet concentration on the relations $\tilde{c}(p, q)$ and $\tilde{\rho}(p, q)$.

In the initial period of operation of the bed, determined by the condition:

$$\alpha\varphi(p, q) \ll 1, \tag{11}$$

$$\tilde{c}(p, q) = [1 + \beta f(q)][1 + \alpha\varphi(p, q)] \exp\left[-p + \int_0^p \tilde{\rho}_i(x/b)\, dx\right]; \tag{12a}$$

$$\tilde{\rho}(p, q) = \tilde{\rho}_i(p) + \alpha\varphi(p, q)[1 - \tilde{\rho}_i(p)] \exp\left[-p + \int_0^p \tilde{\rho}_i(x/b)\, dx\right]. \tag{12b}$$

It is evident that, as a function of p, the character of the behavior of $\tilde{c}(p, q)$ differs depending on the frequency of oscillation f(q). For low frequencies with a short period of oscillation $Q_{min} >> P$ – where P is the length of the filter in dimensionless units – $\tilde{c}(p, q)$ decreases monotonically with an increase in p, i.e., in this case its behavior is the same as for a constant inlet concentration (curve 3 in Fig. 1). However, if the frequency of oscillation is high $Q_{min} < P$, then $\tilde{c}(p, q)$ oscillates along the bed with a decreasing amplitude (curve 4 in Fig. 1). The character of the relation $\tilde{c}(p, q)$ remains the same with an increase in the time of filter operation, the only difference being that the amplitude of the oscillations $\tilde{c}(p, q)$ increases due to a reduction in the filtering quality of the bed.

In contrast to $\tilde{c}(p, q)$, the concentration of the residue $\tilde{\rho}(p, q)$ along the bed does not oscillate with any appreciable amplitude. The reason for the oscillation of $\tilde{\rho}(p, q)$ may be the dependence of the function $\varphi(p, q)$. on p. However, since $\varphi(p, q)$ enters into (8) with the factor $\alpha << 1$, then the effect of $\varphi(p, q)$ on $\tilde{\rho}(p, q)$ is significant only when $\varphi(p, q) \gg 1$. In actual beds, P = 1-5. Thus, the condition $\varphi(p, q) \gg 1$ is equivalent to q >> 1. However, in this case, (1) $\int_{\mu(p)}^{q} f(x)\, dx \to 0$ by virtue of (1), and oscillations are absent (see curves 2 and 3 in Fig. 1). For the same reasons, there are no oscillations in the filtration effect ψ. The dependence of this effect on p is indicated by curve 1 in Fig. 1.

The dependences of $\tilde{c}(p, q)$ and $\tilde{\rho}(p, q)$ on q oscillate for all frequencies. Meanwhile, with an increase in operating time, the pulsations increase to their limiting value: $\tilde{c}_0(q) = 1 + \beta f(q)$, while the amplitude of the pulsations of $\tilde{\rho}(p, q)$ decreases. This reflects the saturation of the bed by impurities. The relations $\tilde{c}(q)$ and $\tilde{\rho}(q)$ are shown by curves 2 and 3 in Fig. 2. Curve 1 illustrates the relation $\psi(q)$. This relation, as $\tilde{\rho}(q)$, is characterized by weak pulsations connected with the decrease in $\varphi(p, q)$, as q increases.

During the final stage of operation of the filter, the following relation is satisfied

$$\alpha q \gg 1, \tag{13}$$

and, in accordance with (1), $\int_{\mu(p)}^{q} f(x)\,dx \to 0,$ while the solutions (8) approach the solutions for a constant inlet concentration equal to $\langle c \rangle_t$. The same applies to the filtration effect.

It should be noted that in the case of a variable inlet concentration, the filtration effect does not give exhaustive information on the protective action of the filter — as it would in the case of a constant inlet concentration. The protective action of the filter for the case of a variable concentration depends on how the filtrate will be used.

If the filtrate is sent to a storage vessel with the exchange time $2\Delta q > Q_{max}$ (maximum period of oscillation), then the time of the protective action of the filter q_p is determined by the relation

$$\frac{1}{2\Delta q}\int_{q-\Delta q}^{q+\Delta q} \tilde{c}(x, P)\,dx \leqslant \tilde{c}_{cr}. \tag{14}$$

Considering (1) and (8), we obtain a relation to determine q_p:

$$\tilde{c}_n(P, q_p) = \tilde{c}_{cr}, \tag{15}$$

where $\tilde{c}_n$ is found from (8a) with $\beta = 0$, i.e., in the case of a constant inlet concentration $\langle c \rangle_t$.

Thus,

$$\tau_p = \frac{1}{\alpha b v}\left\{\ln \frac{\tilde{c}_{cr}}{1-\tilde{c}_{cr}}\left[\exp\left(p - \int_0^p \tilde{\rho}_i(x/b)\,dx\right) - 1\right]\right\}. \tag{16}$$

If instantaneous values of impurity concentration in the filtrate are important for the filtering operation, then the condition for finding q_p changes and, in accordance with (8a), it takes the form

$$\tilde{c}_n(P, q_p)[1 + \beta f(q_p)] \leqslant \tilde{c}_{cr} \tag{17}$$

Since we are interested in the restriction on the amplitude of the impurity concentration in the filtrate, then (15) is changed to the form

$$\tilde{c}_n(P, q_p) = \frac{\tilde{c}_{cr}}{1+\beta}. \tag{18}$$

We use (16) and (18) to find the change in the time of protective action determined from its amplitude relative to the value established on the basis of the mean:

$$\Delta\tau_p = -\frac{1}{\alpha b v}\ln(1+\beta). \tag{19}$$

As the calculation showed, this quantity may be substantial (see Fig. 2). Using $\tau_0 = (\alpha b v)^{-1}$ to designate the time scale of the bed, we use (16) and (19) to obtain the following for the dimensionless time of protective action of the bed

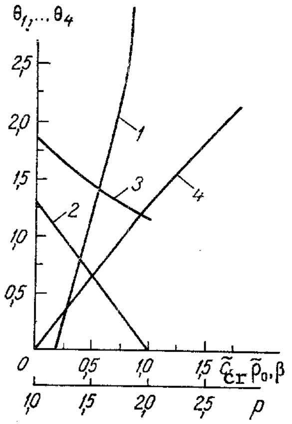

Fig. 3. Dependences of $\theta_p(\tilde{c}_{cr}|\rho_0|\beta|p)$ on $\tilde{c}_{cr}$, $\tilde{\rho}_0$, β and p. Curves 1, 2, 3, and 4 pertain to θ_1, θ_2, θ_3, θ_4, respectively equal to $\theta_p(\tilde{c}_{cr}|0|0.8|2)$, $\theta_p(0.5|\tilde{\rho}_0|\cdot 0.8|2)$, $\theta_p(0.5|0|\beta|2)$ and $\theta_p(0.5|0|0.8|p)$.

$$\theta_p = \frac{\tau_p}{\tau_0} = \ln\left\{\frac{\tilde{c}_{cr}}{1-\tilde{c}_{cr}} \frac{\exp(1-0{,}5\tilde{\rho}_0)-1}{1+\beta}\right\}, \tag{20}$$

where for simplicity we have assumed that the relation $\rho_i(x)$ has the form:

$$\rho_i(x) = \rho_0\left(1-\frac{x}{X}\right). \tag{21}$$

It follows from Eq. (20) that θ_p is a function of $\tilde{c}_{cr}$, $\tilde{\rho}_0$, β and P, which characterize the allowable overshoot concentration, the maximum initial distribution of the residue (quality of regeneration), the amplitude of the oscillations, and the height of the filter bed. The relation $\theta_p(\tilde{c}_{cr}|\tilde{\rho}_0|\beta|P)$, calculated from (20), is shown in Fig. 3. The dependences $\theta_p(\tilde{c}_{cr})$ (curve 1) and $\theta_p(P)$ (curve 4) are the strongest.

The equality of θ_p to zero (curves 1, 2, 4) reflects the unserviceability of the bed at the corresponding values of the parameters. Curves 2 and 3 describe the relations $\theta_p(\rho_0)$ and $\theta_p(\beta)$. It is evident that the quality of regeneration of the bed and the amplitude of oscillations of the inlet concentration also have an appreciable effect on filtration efficiency.

Thus, analysis of the continuous filtration of a suspension with a variable impurity concentration permits the conclusion that the capacity of the bed, filtration efficiency, and the time of protective action — determined from the mean concentration of impurities in the filtrate — nearly coincide with the analogous quantities calculated for the mean inlet concentration.

The time of protective action determined from the allowable instantaneous concentration of impurities in the filtrate may differ significantly from the value found on the basis of the mean inlet concentration if the amplitude of the oscillations is comparable to the mean inlet concentration. Also, the time of protective action may decrease considerably with low-quality regeneration of the bed. For example, it follows from Fig. 3 (curve 2) that at a level of 90% regeneration, the duration of the filtering cycle is reduced more than 12% compared to a pure bed.

NOTATION

C(t), inlet concentration of impurities; <c>, constant component of inlet concentration; c(t), oscillating part of inlet concentration; β, oscillation coefficient; c_{osc}, amplitude of oscillations; f(t), law of oscillation; C(x, t), concentration of impurities at point x of the filter bed at the moment of time t; $\rho(x, t)$, concentration of residue in the bed; $\varepsilon(x, t)$, porosity of bed; v, filtration rate; b, settling parameter characterizing the probability of settling of impurities per unit time; ρ_{cr}, limiting concentration of residue in the bed; $\rho_i(x)$, initial distribution of residue over height of bed; p, dimensionless coordinate; P, height of bed in dimensionless units; q, dimensionless time; Q, period of oscillation; ψ, filtration effect; $\psi = (c(t) - c(x, t))/c(t)$; $\tilde{c}_{cr}$, limitingly allowable concentration of impurities in filtrate; q_D., dimensionless time of protective action of the bed; $\tilde{c}_n(p, q)$, value of $\tilde{c}(p, q)$, corresponding to a constant inlet concentration <c>; τ_p, time of protective action of the bed.

LITERATURE CITED

1. Yu. M. Shekhtman, Filtration of Low-Concentration Dispersions, Moscow (1961).
2. D. M. Mints, Theoretical Principles of Water Purification Technology, Moscow (1964).
3. L. A. Kul'skii and P. P. Strokach, Technology of the Purification of Water [in Russian], Kiev (1986).
4. E. V. Venitsianov and R. N. Rubinshtein, Dynamics of Sorption from Liquids [in Russian], Moscow (1983).

MECHANISM OF MASS TRANSFER OF THE LIQUID PHASE IN THE DISPLACEMENT OF GAS BY LIQUID IN NONUNIFORM POROUS MEDIA

A. I. Ber and V. G. Reitenbakh

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An analysis is made of the possible mechanism of mass transfer in the motion of gas-liquid systems in low-permeability porous media. The mechanism is based on the phenomenon of capillary recondensation.

A considerable part of the gas reserves in the world are confined in low-permeability resevoirs (permeability less than 10^{-3} μm^2). The mechanism of mass transfer determines the motion of gas-liquid systems in media with a low permeability, the conditions for removal of gas and condensate from the bed, the interaction of the drilling-fluid filtrate with the rocks of the bed as it is being opened up by drilling, and other factors. In the case of isothermal two-phase motion, the dominant process of mass transfer is determined by value of the diffusional Peclet number (Pe_d). This number characterizes the ratio of the hydrodynamic and diffusion flows of a substance:

$$\mathrm{Pe}_d = \frac{vL}{D}. \tag{1}$$

In the penetration of a fluid into a gas-saturated porous medium, consideration should be given to diffusion in the length direction of the pores. The characteristic rate can be determined in accordance with the Poiseuille law for laminar motion in a cylindrical capillary

$$v = \frac{r^2}{8\mu}\frac{\Delta P}{L}.$$

In this case, Eq. (1) takes the form

$$\mathrm{Pe}_d = \frac{r^2 \Delta p}{8\mu D}. \tag{2}$$

For low-permeability rock with pores having a radius on the order of 10^{-6}-10^{-8} m in the case of realistic values of the diffusion coefficient 10^{-6}-10^{-8} m^2/sec and viscosity 10^{-3}-10^{-2} Pa·sec and pressure gradients due either to capillary forces or the gradient of the external pressure field at distances commensurate with the size of a block (10^3-10^5 Pa), the number Pe_d will take values on the order of unity or several orders less, i.e., under these conditions the diffusion mechanism of mass transfer is either comparable to the hydrodynamic mechanism or is dominant. This conclusion is backed up by a well-known analysis of moisture exchange processes in low-permeability clayey rocks [1, 2].

The diffusional mechanism of mass transfer may either by diffusion itself or osmotic phenomena considered as a group [1]. We will not concern ourselves here with the latter, since they can take place in the presence of a bound liquid phase either initially present

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